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ALTERNATIVE MODELS INVOLVING MULTIPLE VARIABLES FOR PARTIAL CORRELATION ANALYSES AND THEIR APPLICATIONS

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ABSTRACT

This paper aims at providing various ways of alternating variables so that the number of variables held constant produces the same number of alternative models whose computations give the same result for a particular partial correlation coefficient. Significantly, the numerical verification carried out to show the validity of the alternative models has shown that, in a multiple variable case, there is no one way solution to obtaining a partial correlation coefficient, when the floating effects of others are under control. Hence, we have shown the number of variables held constant in partial correlation gives exact number of alternative ways of providing a result for particular partial correlation coefficient.

INTRODUCTION

Analysis of partial correlation is an analysis which involves more than two variables. The analysis of partial correlation was introduced in an attempt to measure the extent to which two variables associate, while the floating effects of other variables under consideration are held constant. The full meaning and implication of the concept of partial correlation can be grasped with the knowledge of multiple linear regression analysis. In a multiple relationship among three variables, X_1, X_2 and X_3, three partial correlations are possible. They are the partial correlation of X_1 and X_2, holding X_3 constant, the partial correlation of X_1 and X_3, holding X_2 constant, and partial correlation of X_2 and X_3, holding X_1 constant, Olayemi and Olayide (1981). Sometime the correlation between two variables say X_1 and X_2 may be partly due to the correlation of a group of variables $X_3,\ldots,X_p$ with both X_1 and X_2. In such situations, one may want to know what the correlation between X_1 and X_2 would be if the effects of $X_3 \ldots X_p$ on each of them were eliminated. This is called partial correlation or net correlation between X_1 and X_2, Onyeagu (2003).

Spiegel (1992) expressed models for partial correlation analysis with a total of four variables; a dependent and three independent variables. He alternated two variables held constant for a partial correlation analysis. His models were limited to only three independent variables in the partial correlation analyses. Johnston and Dinardo (1997) stated partial correlation model between a dependent variable 'Y' and independent variable 'X_2' conditioned on $X_3,\ldots,X_k$ as

$$r_{12\cdot3\ldots k} = \frac{X_2^1 M^* y}{\sqrt{X_2^1 M^* X_2 y^1 M^* y}},$$

Where M^*y is the vector residuals when y is regressed on X and M^*X_2 is the vector of residuals when X_2 is regressed on X. M is symmetric, indempotent matrix with the property $M^*X = 0$. Gujarati (1995) obtained correlation coefficients of order p in terms of correlation coefficients of order p-1 as

$$r_{12\cdot345\ldots p} = \frac{r_{12\cdot345\ldots(p-1)} - [r_{1p.345\ldots(p-1)} r_{2p.345\ldots(p-1)}]}{\sqrt{[1-r^2_{1p.345\ldots(p-1)}][1-r^2_{2p.345\ldots(p-1)}]}}$$

Gujarati failed to introduce alternative models whose numerical computations give the same result for $r_{12\cdot345\ldots p}$ as stated above. Using data for Total Tax Revenue (X_1), Petroluem Profit Tax (X_2), Company Income Tax (X_3), Education Tax(X_4), Personal Income Tax (X_5) and Value Added Tax (X_6) obtained from Federal Inland Revenue for the period (1990-2002), alternative models for $r_{12\cdot345\ldots p}$ and other partial correlation models for the variables constitute this work.

GENERALIZED MODELS

(A) Partial correlation models for a fixed dependent variable X_1 and independent variable X_2, while others are held constant are developed as follows:

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$$R_{12.3456\ldots k} = \frac{R_{12.456\ldots k} - R_{13.456\ldots k}R_{23.456\ldots k}}{\sqrt{[1- R^2_{13.456\ldots k}][1- R^2_{23.456\ldots k}]}}$$

$$= \frac{R_{12.356\ldots k} - R_{14.356\ldots k}R_{24.356\ldots k}}{\sqrt{[1- R^2_{14.356\ldots k}][1- R^2_{24.356\ldots k}]}}$$

$$= \frac{R_{12.346\ldots k} - R_{15.346\ldots k}R_{25.346\ldots k}}{\sqrt{[1- R^2_{15.346\ldots k}][1- R^2_{25.346\ldots k}]}}$$

$$= \frac{R_{12.345\ldots k} - R_{16.345\ldots k}R_{26.345\ldots k}}{\sqrt{[1- R^2_{16.345\ldots k}][1- R^2_{26.345\ldots k}]}}$$

$$\vdots \qquad\qquad \vdots$$

$$= \frac{R_{12.3456\ldots k} - R_{1k.3456\ldots k-1}R_{2k.3456\ldots k-1}}{\sqrt{[1- R^2_{1k.3456\ldots k-1}][1- R^2_{2k.3456\ldots k-1}]}}$$

(B) Partial correlation models for a fixed dependent variable X_1 and independent variable X_3 while others are held constant are developed as follows

$$R_{13.2456\ldots k} = \frac{R_{13.456\ldots k} - R_{12.456\ldots k}R_{23.456\ldots k}}{\sqrt{[1- R^2_{12.456\ldots k}][1- R^2_{23.456\ldots k}]}}$$

$$= \frac{R_{13.256\ldots k} - R_{14.256\ldots k}R_{34.256\ldots k}}{\sqrt{[1- R^2_{14.256\ldots k}][1- R^2_{34.256\ldots k}]}}$$

$$= \frac{R_{13.246\ldots k} - R_{15.246\ldots k}R_{35.246\ldots k}}{\sqrt{[1- R^2_{15.246\ldots k}][1- R^2_{35.246\ldots k}]}}$$

$$= \frac{R_{13.245\ldots k} - R_{16.245\ldots k}R_{36.245\ldots k}}{\sqrt{[1- R^2_{16.245\ldots k}][1- R^2_{36.245\ldots k}]}}$$

$$\vdots \qquad\qquad \vdots$$

$$= \frac{R_{13.2456\ldots k} - R_{1k.2456\ldots k-1}R_{3k.2456\ldots k-1}}{\sqrt{[1- R^2_{1k.2456\ldots k-1}][1- R^2_{3k.2456\ldots k-1}]}}$$

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(C) Partial correlation models for a fixed dependent variable X_1 and independent variable X_4 while others are held constant are developed as follows

$$\begin{aligned}
R_{14\cdot 2356\ldots k} &= \frac{R_{14\cdot 356\ldots k} - R_{12\cdot 356\ldots k}R_{24\cdot 356\ldots k}}{\sqrt{[1- R^2_{12\cdot 356\ldots k}][1- R^2_{24\cdot 356\ldots k}]}} \\
&= \frac{R_{14\cdot 256\ldots k} - R_{13\cdot 256\ldots k}R_{34\cdot 256\ldots k}}{\sqrt{[1- R^2_{13\cdot 256\ldots k}][1- R^2_{34\cdot 256\ldots k}]}} \\
&= \frac{R_{14\cdot 236\ldots k} - R_{15.236\ldots k}R_{45\cdot 236\ldots k}}{\sqrt{[1- R^2_{15\cdot 236\ldots k}][1- R^2_{45\cdot 236\ldots k}]}} \\
&= \frac{R_{14\cdot 235\ldots k} - R_{16\cdot 235\ldots k}R_{46\cdot 235\ldots k}}{\sqrt{[1- R^2_{16\cdot 235\ldots k}][1- R^2_{46\cdot 235\ldots k}]}} \\
& \quad \vdots \\
&= \frac{R_{14\cdot 2356\ldots k} - R_{1k\cdot 2356\ldots k-1}R_{4k\cdot 2356\ldots k-1}}{\sqrt{[1- R^2_{1k\cdot 2356\ldots k-1}][1- R^2_{4k\cdot 2356\ldots k-1}]}}
\end{aligned}$$

(D) Partial correlation models for a fixed dependent variable X_1 and independent variable X_5 while others are held constant are developed as follows

$$\begin{aligned}
R_{15\cdot 2346\ldots k} &= \frac{R_{15\cdot 456\ldots k} - R_{12\cdot 346\ldots k}R_{25\cdot 346\ldots k}}{\sqrt{[1- R^2_{12\cdot 346\ldots k}][1- R^2_{25\cdot 346\ldots k}]}} \\
&= \frac{R_{15\cdot 246\ldots k} - R_{13\cdot 246\ldots k}R_{35\cdot 246\ldots k}}{\sqrt{[1- R^2_{13\cdot 246\ldots k}][1- R^2_{35\cdot 246\ldots k}]}} \\
&= \frac{R_{15\cdot 236\ldots k} - R_{14.236\ldots k}R_{45\cdot 236\ldots k}}{\sqrt{[1- R^2_{14\cdot 236\ldots k}][1- R^2_{45\cdot 236\ldots k}]}} \\
&= \frac{R_{15\cdot 234\ldots k} - R_{16\cdot 234\ldots k}R_{56\cdot 234\ldots k}}{\sqrt{[1- R^2_{16\cdot 234\ldots k}][1- R^2_{56\cdot 234\ldots k}]}} \\
& \quad \vdots \\
&= \frac{R_{15\cdot 2346\ldots k} - R_{1k\cdot 2346\ldots k-1}R_{5k\cdot 2346\ldots k-1}}{\sqrt{[1- R^2_{1k\cdot 2346\ldots k-1}][1- R^2_{5k\cdot 2346\ldots k-1}]}}
\end{aligned}$$

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(E) Partial correlation models for a fixed dependent variable X_1 and independent variable X_6 while others are held constant are developed as follows

$$R_{16.2345\ldots k} = \frac{R_{16.345\ldots k} - R_{12.345\ldots k}R_{26.345\ldots k}}{\sqrt{[1- R^2_{12.345\ldots k}][1- R^2_{26.345\ldots k}]}}$$

$$= \frac{R_{16.245\ldots k} - R_{13.245\ldots k}R_{36.245\ldots k}}{\sqrt{[1- R^2_{13.245\ldots k}][1- R^2_{36.245\ldots k}]}}$$

$$= \frac{R_{16.235\ldots k} - R_{14.235\ldots k}R_{46.235\ldots k}}{\sqrt{[1- R^2_{14.235\ldots k}][1- R^2_{46.235\ldots k}]}}$$

$$= \frac{R_{16.234\ldots k} - R_{15.234\ldots k}R_{56.234\ldots k}}{\sqrt{[1- R^2_{15.234\ldots k}][1- R^2_{56.234\ldots k}]}}$$

$$\vdots$$

$$= \frac{R_{16.2345\ldots k} - R_{1k.2345\ldots k-1}R_{6k.2345\ldots k-1}}{\sqrt{[1- R^2_{1k.2345\ldots k-1}][1- R^2_{6k.2345\ldots k-1}]}}$$

The above models continue up to k^{th} independent variable. The models for k^{th} variable are

$$R_{1k.23456\ldots k} = \frac{R_{1k.3456\ldots k-1} - R_{12.3456\ldots k-1}R_{2k.3456\ldots k-1}}{\sqrt{[1- R^2_{12.3456\ldots k-1}][1- R^2_{2k.345\ldots k-1}]}}$$

$$= \frac{R_{1k.2456\ldots k-1} - R_{13.2456\ldots k-1}R_{3k.2456\ldots k-1}}{\sqrt{[1- R^2_{13.2456\ldots k-1}][1- R^2_{3k.2456\ldots k-1}]}}$$

$$= \frac{R_{1k.2356\ldots k-1} - R_{14.2356\ldots k-1}R_{4k.2356\ldots k-1}}{\sqrt{[1- R^2_{14.2356\ldots k-1}][1- R^2_{4k.2356\ldots k-1}]}}$$

$$= \frac{R_{1k.2346\ldots k-1} - R_{15.2346\ldots k-1}R_{5k.2346\ldots k-1}}{\sqrt{[1- R^2_{15.2346\ldots k-1}][1- R^2_{5k.2346\ldots k-1}]}}$$

$$\vdots$$

$$= \frac{R_{1k.23456\ldots k-2} - R_{1(k-1).23456\ldots k-2}R_{k(k-1).23456\ldots k-2}}{\sqrt{[1- R^2_{1(k-1).23456\ldots k-2}][1- R^2_{k(k-1).23456\ldots k-2}]}}$$

The above models are generalized models for partial correlation analyses.

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NUMERICAL VERIFICATION OF THE MODELS

The partial correlation results for only three variables held constant are provided in the table 1:

TABLE 1: Partial correlation results with three variables held constant

$R_{12\cdot456} = 0.9804$	$R_{13\cdot256} = 0.1366$	$R_{12\cdot345} = 0.8760$	$R_{15\cdot234} = 0.6380$
$R_{13\cdot456} = 0.4480$	$R_{14\cdot256} = -0.6820$	$R_{16\cdot345} = 0.2068$	$R_{16\cdot234} = 0.7128$
$R_{23\cdot456} = 0.4468$	$R_{34\cdot256} = -0.1400$	$R_{26\cdot345} = -0.2419$	$R_{56\cdot234} = 0.0369$
$R_{12\cdot346} = 0.9898$	$R_{13\cdot246} = -0.1994$	$R_{13\cdot245} = 0.3769$	
$R_{15\cdot346} = 0.9486$	$R_{15\cdot246} = 0.8777$	$R_{16\cdot245} = 0.9108$	
$R_{25\cdot346} = 0.9050$	$R_{35\cdot246} = -0.257$	$R_{36\cdot245} = 0.3909$	
$R_{12\cdot356} = 0.9557$	$R_{14\cdot236} = -0.1862$	$R_{14\cdot235} = 0.2975$	
$R_{14\cdot356} = -0.1582$	$R_{15\cdot236} = 0.7592$	$R_{16\cdot235} = 0.8170$	
$R_{24\cdot356} = 0.0425$	$R_{45\cdot236} = 0.3063$	$R_{46\cdot235} = 0.7030$	

COMPUTATION:

(i) $$R_{12\cdot3456\ldots k} = \frac{0.9804 - (0.4480)(0.4468)}{\sqrt{[1-(0.4480)^2[1-(0.4468)^2]}} = 0.98$$

$$= \frac{0.9557 - (-0.1582)(0.0425)}{\sqrt{[1-(-0.1582)^2[1-(0.0425)^2]}} = 0.98$$

$$= \frac{0.9898 - (0.9486)(0.9050)}{\sqrt{[1-(0.9486)^2[1-(0.9050)^2]}} = 0.98$$

$$= \frac{0.8760 - (0.2068)(-0.2419)}{\sqrt{[1-(0.2068)^2[1-(-0.2419)^2]}} = 0.98$$

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Hence, the above models are verified correct for $R_{12\cdot3456\ldots k}$

(ii) $$R_{13\cdot2456\ldots k} = \frac{0.4480 - (0.9804)(0.4468)}{\sqrt{[1-(0.9804)^2[1-(0.4468)^2]}} = 0.06$$

$$= \frac{0.3769 - (0.9110)(0.3901)}{\sqrt{[1-(0.9110)^2[1-(0.3901)^2]}} = 0.06$$

$$= \frac{-0.1994 - (0.8777)(-0.2571)}{\sqrt{[1-(0.8777)^2[1-(-0.2571)^2]}} = 0.06$$

$$= \frac{0.3770 - (0.9108)(0.3909)}{\sqrt{[1-(0.9108)^2[1-(0.3909)^2]}} = 0.06$$

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Hence, the above models are verified correct for $R_{13\cdot2456\ldots k}$

(iii) $R_{14\bullet 2356\ldots k} = \dfrac{-0.1582 - (0.9557)(0.0425)}{\sqrt{[1-(0.9557)^2[1-(0.0425)^2]}} = -0.68$

$= \dfrac{-0.6820 - (0.1366)(-0.1400)}{\sqrt{[1-(0.1366)^2[1-(-0.1400)^2]}} = -0.68$

$= \dfrac{-0.1862 - (0.7592)(0.3063)}{\sqrt{[1-(0.7592)^2[1-(0.3063)^2]}} = -0.68$

$= \dfrac{0.2975 - (0.8170)(0.7030)}{\sqrt{[1-(0.8170)^2[1-(0.7030)^2]}} = -0.68$

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Hence, the above models are verified correct for $R_{14\bullet 2356\ldots k}$

(iv) $R_{15\bullet 2346\ldots k} = \dfrac{0.9486 - (0.9898)(0.9050)}{\sqrt{[1-(0.9898)^2[1-(0.9050)^2]}} = 0.87$

$= \dfrac{0.8777 - (-0.1994)(-0.257)}{\sqrt{[1-(-0.1994)^2[1-(-0.257)^2]}} = 0.87$

$= \dfrac{0.7592 - (-0.1862)(0.3063)}{\sqrt{[1-(-0.1862)^2[1-(0.3063)^2]}} = 0.87$

$= \dfrac{0.6380 - (0.7128)(0.0369)}{\sqrt{[1-(0.7128)^2[1-(0.0369)^2]}} = 0.87$

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Hence, the above models are verified correct for $R_{15\bullet 2346\ldots k}$

(v) $R_{16\bullet 2345\ldots k} = \dfrac{0.2068 - (0.8760)(-0.2419)}{\sqrt{[1-(0.8760)^2[1-(-0.2419)^2]}} = 0.89$

$= \dfrac{0.9108 - (0.3770)(0.3909)}{\sqrt{[1-(0.3770)^2[1-(0.3909)^2]}} = 0.89$

$= \dfrac{0.8170 - (0.2975)(0.7030)}{\sqrt{[1-(0.2975)^2[1-(0.703)^2]}} = 0.89$

$$= \frac{0.7128 - (0.6380)(0.0369)}{\sqrt{[1-(0.6380)^2[1-(0.0369)^2]}} = 0.89$$

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Hence, the above models are verified correct for $R_{16\cdot2345\ldots k}$

CONCLUSION

Partial correlation coefficient involving many variables and various ways of alternating the variables to give the same result for each correlation coefficient was our study interest. The achievement recorded in this work is the alternation of all the variables held constant without causing alteration to each result. Significantly, the re-arrangement of variables which provides various models for a particular partial correlation coefficient explains the equivalent importance of all the variables under control. This ascertains the fact the number of variables held constant provides exact number of alternative models for the same partial correlation coefficient.

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