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COMPARATIVE STUDY OF QUARTERLY RAINFALL IN CROSS RIVER STATE THROUGH APPLICATION OF SEASONAL AUTOREGRESSIVE INTEGRATED MOVING AVERAGE MODEL: A CASE OF CALABAR AND OGOJA

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ABSTRACT

This paper seeks to fit seasonal models to quarterly rainfall data in two extreme areas of Cross River State; Calabar and Ogoja areas, represented by X, and Y, respectively. The two series exhibit pure seasonal model. The ACF and PACF of the seasonally differential series suggested SARIMA (0, 0, 0) x (4, 1, 1)$_4$ for each of the series. The models exhibit pure white noise and stationarity conditions. The residuals of the two models exhibit pure white noise process. Akaike's Information Criterion (AIC) adopted indicates a minor difference of 0.55 between the two models. This further establishes the fact that significant difference in the amount of rainfall between the two extreme areas of Cross River State does not controvert the seasonal order of the two models.

INTRODUCTION

The fluctuations seen in a time series can be classified as repeatable or non-repeatable. Seasonality can be defined as a pattern of a time series, which repeats at regular intervals every year. In evaluating whether the economy or particular aspects of the economy is in a business cycles is a fundamental task. Seasonally adjusted data providing more interpretable measures of changes occurring in a given period reflects real economic movements without the misleading seasonal changes. A time series from which the seasonal movements have been eliminated allows the comparison of data between two months or quarters for which the seasonal pattern is different, Oguz and Beyza (2002).

THE STUDY AREA

Cross River State is a state in the South-South geopolitical zone in Nigeria. The state is bounded by Benue State and Taraba in the North, Akwa Ibom State in the west, Cameroon in the East and Atlantic Ocean in the South. The distance between the two Local Government Areas in the State is about 280km. The motivation behind the choice of the two distant areas is that Calabar experience heavy rainfall in the season than Ogoja. In Ogoja, the short period of rainfall comes with heavy wind. The Local Government, like in the North experiences more of summer than winter.

PURE SEASONAL TIME SERIES MODEL

Tsay (2009) stated general model,

$\Phi(B^s)Z_t = C + \Theta(B^s)d_t$, where C is constant,

$\Phi(B^s) = 1 - \Theta_1 B^s - \Phi_2 B^{2s} - \ldots - \Phi_{12} B^{ps}$,

$\Theta(B^s) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} - \ldots - \Theta_Q B^{Qs}$

A pure seasonal ARIMA (1,1,1,) model of period 12 is

$(1 - \Phi_2 B^{12})Z_t = (1 - \Theta B^{12})a_t$,

While that of period 4 is $(1 - \Phi_2 B^4)Z_t = (1 - \Theta B^4)a_t$,

The generalized form is SARIMA model can be written as (Box et al., 2008; Cryer and Chan, 2008):

$\varphi p(B)\Phi_p(B^s)(1-B)^d(1-B^s)^D Z_t = \Theta_q(B)\Theta_Q(B^S)a_t$

Where: $\varphi p(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \text{---} - \varphi_1 B^P$, the polynomials of the non-seasonal autoregressive part,

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$\Phi_P(B^S) = 1 - \Theta_1 B^S - \Theta_2 B^{2S} - \text{---} - \Phi_P B^{PS}$, the polynomials of the seasonal autoregressive part,

$\Theta_q(B) = 1 - \Theta_1 B - \Theta_2 B^q - \text{---} - \Theta_q B^q$, the polynomials of the non-seasonal moving average part,

$\Theta_Q(B^S) = 1 - \Theta_1 B^S - \Theta_2 B^{2S} - \text{---} - \Theta_Q B^{QS}$, the polynomials of the non-seasonal moving average part,

Where:
B = the backward shift operator
d and D = the non-seasonal and seasonal order of difference, respectively and usually abbreviated as SARIMA $(p,d,q)(P,D,Q)_s$.

When there is no seasonal effect, a SARIMA model reduce to pure ARIMA (p,d,q) and when the time series dataset is stationary a pure ARIMA reduces to ARMA (p,q).

RELATED WORK

To date, SARIMA model has been used in various fields of forecasting. For example, haswell et al. (2003) applied this model for forecasting soil dryness index in the southwest of western Australia; Hu et al (2004) predicted Ross River virus disease. Modarres (2007), also Abebe and Foerch (2008) for drought forecasting; Ediger et al (2006), also Ediger and Akar (2007) for forecasting production of fossil fuel sources in Turkey: Briet et al. (2008) for short term malaria prediction in Sri Lanka; Sumer et al. (2009) for forecasting electricity demand; Chen et al. (2009) for forecasting inbound air travel arrivals to Taiwan; Abraham et al. (2009) for short-term forecasting of emergency inpatient flow; Schulze and Prinz (2009) for forecasting container transshipment in Germany, Momani (2009) for forecasting rainfall in Jordan; Ibrahim et al. (2009) for air pollutants prediction in several area of Malaysia. More recently, Pozza et al. (2010) applied SARIMA to analysis of PM2.5 and PM10-25 mass concentration in the city of Carlos.

Ong et al. (2005) stated that although many previous papers have concentrated on model estimation, model identification is actually the most crucial stage in building ARIMA model, because false model identification will cause the wrong stage of model estimation and increase the cost of re-identification. In particular of SARIMA models, most of previous papers usually used directly the multiplicative model without testing whether the multiplicative parameter was significant. It means that the multiplicative SARIMA models assume that there is a significant parameter as a result of multiplicative between non-seasonal and seasonal parameter. Moreover, most popular statistical software such as MINITAB and SPSS only have facility to fit a multiplicative model.

Haper (1991) stated that when examining figures subjected to seasonal variation, one of the aims is to know whether some other factors are involved. If the seasonal variations are known, then aim to remove the seasonal influences from the figures. Hence the resulting figures are said to be deseasonalized. Suhartono (2011) applied time series forecasting by using seasonal autoregressive integrated moving average: subset, multiplicative or additive model. Helman (2011) offered an analysis of monthly average temperature and precipitation sum time series recorded at 44 measurement stations in the Chec Republic over the period of 1961-2008. The two objectives to be achieved were the construction of SARIMA models based on Box-Jenkins methodology and a comparison of different models constructed according to the given factors of particular measurement stations; elevation longitude and latitude. Zongwu and Chen (2005) proposed a new class of flexible seasonal time series models to characterize the trend and seasonal variations. The proposed model consist of a common trend function over periods and additive individual trend (seasonal effect) functions that are specific to each season within periods. A local linear approach was developed to estimate the trend and seasonal effect functions. The consistency and asymptotic normality of the proposed estimators, together with a consistent estimator of the asymptotic variance are obtained under the a-mixing condition and without specifying the error distribution. The proposed methodologies are illustrated with a simulated example and two economic and financial time series, which exhibit nonlinear and non-stationary behavior.

In this work, the interest is not only to fit models to the two sets of seasonal (rainfall) data. The motivation is to compare the seasonal patterns of the rainfall data in Calabar and Ogoja Local Government Areas, so as to find out whether the changes in the amount of rainfall in the two areas for the period under study have effect on the type of seasonal models for the sets of data.

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MODEL IDENTIFICATION

The plots of autocorrelation and partial autocorrelation functions of the seasonally differenced series suggested SARIMA (4, 1, 1) for each of the two sets of data. That means autoregressive part of the seasonal series is of order "4" while the moving average part is of order "1" for each series.

ESTIMATION OF PARAMETERS OF SARIMA MODELS

SARIMA MODEL FOR X_t

The proposed seasonal Autoregressive integrated moving Average model for the analysis of X_t is SARIMA $(0,0,0)x\ (4,1,1)_4$. This is further expressed at $(1 - \Phi_4B^4 - \Phi_8B^8 - \Phi_{12}B^{12} - \Phi_{16}B^{16})(1 - B^4)X_t = (1 - \Phi_4B^4)e_t$ 4.1

The expansion of the above model is as shown in section '2' of chapter '3'. The first part of the model is non-seasonal while the second is the seasonal part. Estimates of the parameters as obtained with statistical software are given in the table 4.1.

Table 4.1: Estimates of the Parameters of X_t

SARIMA	Coeff	St. Dev.	T.
SAR 4	-0.1483	0.4243	-0.35
SAR 8	-0.2013	0.3099	-0.65
SAR 12	-0.1046	0.2490	-0.42
SAR 16	-0.152	0.2030	0.70
SMA 4	0.5999	0.4316	1.39

Sarima Model for Y_t

The proposed seasonal Autoregressive integrated moving Average model for the analysis of Y_t is SARIMA $(0,0,0)x(4,1,1)_4$. This is further expressed as

$(1 - \Phi_4B^4 - \Phi_8B^8 - \Phi_{12}B^{12} - \Phi_{16}B^{16})(1 - B^4)Y_t = (1 - \Phi_4B^4)e_t$ . . . 4.2

The expansion of the above model is as shown in chapter '3'. The first is non-seasonal, while the second part is seasonal.

Estimates of the Parameter are given in Table 4.2

SARIMA	Coeff	St. Dev.	T.
SAR 4	-0.1114	0.1076	-0.11
SAR 8	-0.1277	0.1098	-1.16
SAR 12	-0.2547	0.1091	-2.34
SAR 16	-0.1225	0.1157	-1.06
SMA 4	0.9775	0.0451	21.69

AKAIKE'S INFORMATION CRITERION

AIC = (2k/n) + ln(RSS/n),

Where RSS = residuals sum of squares

K = number of parameters in the model

N= number of residual observations

From the estimates, AIC for model X_t = 10.450.

AIC for model Y_t = 9.901.

This information provides insignificant difference of 0.55 between the models.

CONCLUSION

It is obvious that the two seasons; rainy and dry seasons are observed concomitantly in different parts of Nigeria. The question as to whether variations in the amounts of rainfall in different part of Cross River State would have influence on seasonal model motivated the research interest. Two extreme areas of Cross River State; Calabar with heavy rainfall and Ogoja with much more of wind blows than rainfall were selected, and are represented by X_t and Y_t respectively. The analysis carried has shown that difference in the amounts of rainfall in the two areas of Cross River State does not affect the pattern of the seasonal model for the two series.

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